●●○○○ **Level 2 · Mathematics** **M2.4**

Fractions

Fair shares, parts of a whole, and numbers between numbers.

Learn what a fraction is by sharing, folding and drawing. Then put fractions on a number line, compare them, and add them.

YOU ARE READY FOR THIS BOOK IF

You can share 12 beans equally into 3 cups and say how many are in each.

One step up	M3.3 Fractions, decimals and percentages
This book	**M2.4 Fractions**
One step down	M1.4 Fair shares and halves

Contents

M2.4.1 Fair shares: equal parts

M2.4.2 One part: halves, quarters, thirds, fifths

M2.4.3 More than one part

M2.4.4 Fractions of a group

M2.4.5 The number line, past 1

M2.4.6 Same amount, different names

M2.4.7 Which is bigger

M2.4.8 Adding and taking away parts

Key words, A to Z

Written and drawn by attnom. Free to read, print and share.

M2.4.1 CHAPTER 1

Fair shares: equal parts

My brother cut the bread in two. Is his piece really a half?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Yes. There are two pieces, so each one is a half.
- ☐ **B** Only if the two pieces are the same size.
- ☐ **C** Only if he cut it right down the middle line.

BY THE END YOU CAN

- Tell if a shape is cut into equal parts
- Share a group of things fairly between 2, 3 or 4 people
- Explain why two pieces of different sizes are not halves

Two friends share one flat bread. They cut it into two pieces. One piece is big. One piece is small. Is that fair?

A share is fair when every piece is the same size. We call these pieces **equal parts**.

DRAW IT FIRST

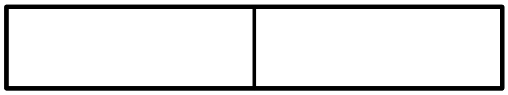
Cut *a* strip into two equal parts

one whole



1 Draw *a* long strip. The whole strip is one **whole**.

one whole



2 Fold it so the two ends meet. Draw *a* line on the fold.

one whole



one half

3 Shade one part. It is one **half** of the strip.



halves



not halves

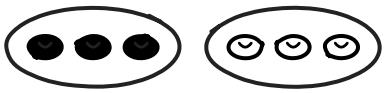
Both bars have two parts. Only the top bar has two equal parts.

SEE IT THREE WAYS

With things

Put 6 beans in a line. Give one to each of 2 people in turn. Stop when none are left.

As a picture



In symbols

6 ÷ 2 = 3. Each person gets 3.

TRY IT

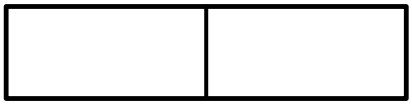
Fold a fair share

You need: a sheet of paper, or an old page, a pencil.

1. Fold the paper so that its edges meet.
2. Press the fold flat, then open it.
3. Draw along the fold. Now fold it again and look.

What you will see: One half lies exactly on top of the other. Nothing sticks out. That is how you know the two parts are equal.

If you cannot try it, this is what happens.



two equal halves

A WRONG IDEA MANY PEOPLE HAVE

~~Any two pieces are two halves.~~

Why it feels right: People say "cut it in half" for any cut into two pieces.

The test that breaks it: Cut a bread into one big piece and one small piece. Ask your friend to pick. Nobody picks the small one.

What is true: Halves are always equal. Two pieces of different sizes are just two pieces.

KEY WORDS

whole
all of one thing, before it is cut.

equal parts
parts that are all the same size.

half
one of two equal parts of a whole.

BACK TO YOUR GUESS

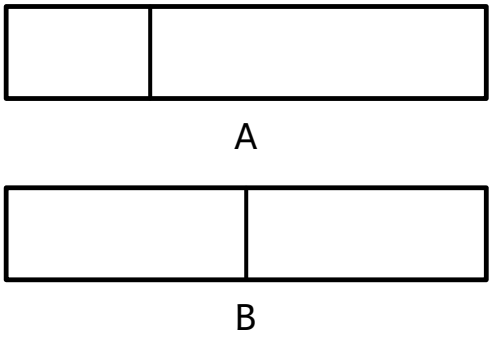
The answer is B: Only if the two pieces are the same size.

Two pieces are halves only when they are the same size. A cut down the middle is one way, not the only way. The sizes are what count.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Which bar is cut into halves?



   A Bar A

   B Bar B

   Answer at the back of the book.

2. Share 12 stones fairly between 3 friends. How many stones does each friend get?

   Answer at the back of the book.

3. Ravi has 10 beans. He gives 6 to his sister and keeps 4. Is that a fair share?

   A Yes, they both have some.

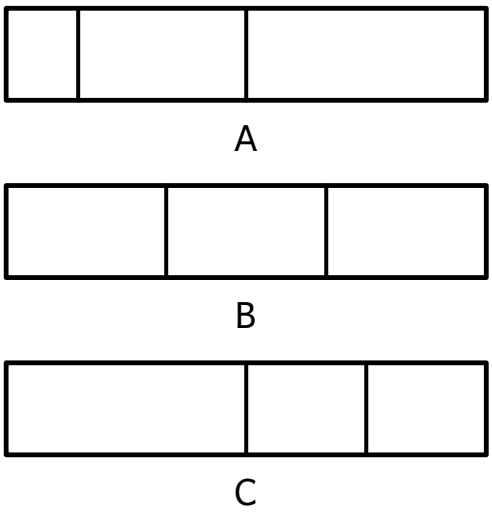
   B No. A fair share is 5 each.

   Answer at the back of the book.

4. Draw a square. Cut it into 4 equal parts. Then find a second way to do it.

   Answer at the back of the book.

5. Tariq wants 3 equal parts of a bread. Which bar shows 3 equal parts?



   A Bar A

   B Bar B

   C Bar C

   Answer at the back of the book.

6. Amina shares 12 beans fairly between 4 friends. How many beans does each friend get?

   A 4 beans each

   B 8 beans each

   C 3 beans each

   Answer at the back of the book.

7. Sipho has 15 beans for 3 children. Which plan is a fair share?

   A Give 5 beans to each child.

   B Give 7, 5 and 3 beans.

   C Give 3 beans to each child.

   Answer at the back of the book.

8. Leila cuts a bread into pieces to share. How can she check that the pieces are equal?

   A Count the pieces.

   B Check that every cut is a straight line.

   C Lay one piece on top of another. They match exactly.

   Answer at the back of the book.

9. Mei cuts a bread into two parts, as in the picture. She says each part is a half. What do you say?



bread cut in two

   A Yes. There are two parts, so they are halves.

   B No. The two parts are not the same size.

   C Yes. Together the two parts make the whole bread.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Equal parts are the same size. A half is one of two equal parts. Two pieces are only halves if they are equal.

Draw: A strip cut into two equal parts. Shade one part and write "one half".

M2.4.2 CHAPTER 2

One part: halves, quarters, thirds, fifths

Eight is more than four. So is $^1/_8$ more than $^1/_4$?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Yes. 8 is more than 4, so one eighth is more.
- ☐ **B** No. One eighth is a smaller part than one quarter.
- ☐ **C** They are the same size.

BY THE END YOU CAN

- Name one equal part as a half, quarter, third or fifth
- Say what the bottom number of a fraction tells you
- Tell which of two parts is smaller, such as $^1/_8$ and $^1/_4$

Last time you cut a whole into equal parts. Now we give one of those parts a name.

DRAW IT FIRST

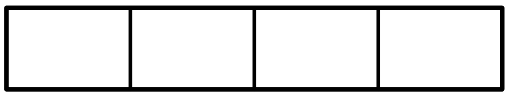
Name one part of *a* strip

one whole



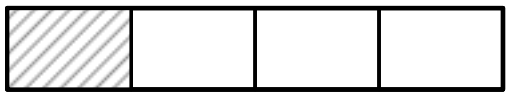
1 Draw *a* long strip. The whole strip is one whole.

one whole



2 Cut it into 4 equal parts. Fold it twice to be sure.

one whole



$\frac{1}{4}$ **one quarter**

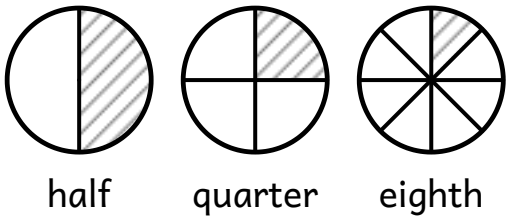
3 Shade one part. It is one **quarter** of the strip, *and* we write $^1/_4$.

The 4 on the bottom is the **bottom number**. It tells how many equal parts the whole is cut into.

The 1 on top says we take one part. $^1/_4$ is *a* fraction. A fraction is *a* number for part of *a* whole.

Some people say fourth, not quarter. Both names mean one of 4 equal parts.

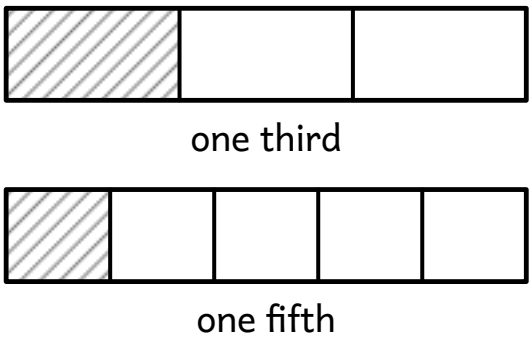
Cut *a* whole into 2 equal parts. One part is one half, $^1/_2$. Cut it into 8 equal parts. One part is one eighth, $^1/_8$.



One part of each circle is shaded.

Thirds and fifths are clearer on a bar. A circle is hard to cut into 3 or 5 equal parts. Cut a bar into 3 equal parts. Each part is a **third**, $^1/_3$.

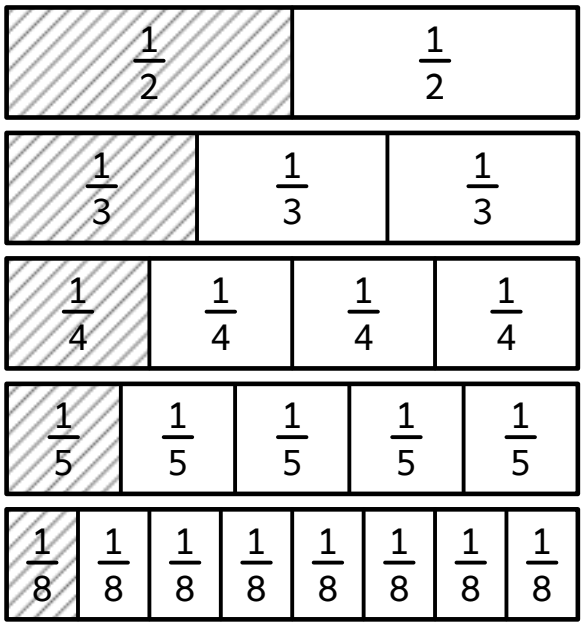
Cut a bar into 5 equal parts. Each part is a **fifth**, $^1/_5$.



The two bars are the same length.

Now look at the bottom numbers. 8 is more than 4. But one eighth is a smaller part than one quarter.

Why? The more parts you cut, the smaller each part is.



Going down, the bottom number grows and the shaded part shrinks.

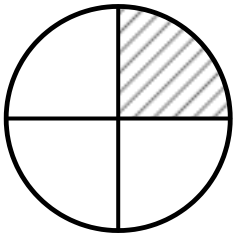
So $^1/_8$ is smaller than $^1/_4$. And $^1/_4$ is smaller than $^1/_2$.

SEE IT THREE WAYS

With things

Fold a paper into 4 equal parts. Open it. Colour one part.

As a picture



In symbols

1 part of 4 equal parts is $^1/_4$.

TRY IT

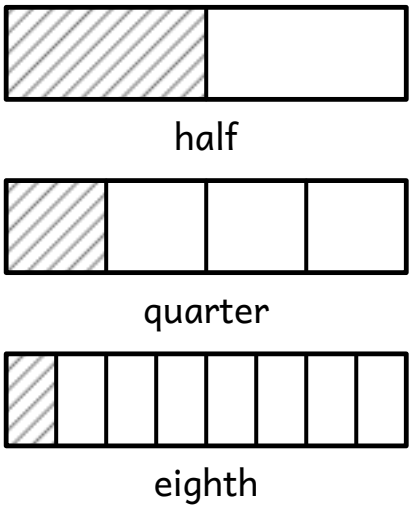
Fold and look at one part

You need: a sheet of paper, or an old page, a pencil.

1. Fold the paper in half. Open it. You have 2 equal parts. Look at one part.
2. Fold it in half, then in half again. Open it. You have 4 equal parts.
3. Fold it in half 3 times. Open it. You have 8 equal parts.
4. Colour one part each time. Compare how big each part is.

What you will see: The more times you fold, the more parts you get. Each part is smaller. One eighth is the smallest part.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~The bigger the bottom number, the bigger the part.~~

Why it feels right: Bigger numbers usually mean more. 8 is more than 4.

The test that breaks it: Take two breads of the same size. Cut one into 4 equal parts. Cut the other into 8. Pick one part from each. Which part is bigger?

What is true: The bottom number counts the parts. More parts means smaller parts. So $^1/_8$ is smaller than $^1/_4$.

KEY WORDS

bottom number
the number under the line. It tells how many equal parts the whole is cut into.

quarter
one of 4 equal parts of a whole. It is written $^1/_4$. Also called a fourth.

third
one of 3 equal parts of a whole. It is written $^1/_3$.

fifth
one of 5 equal parts of a whole. It is written $^1/_5$.

BACK TO YOUR GUESS

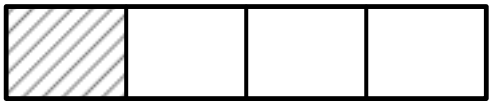
The answer is B: No. One eighth is a smaller part than one quarter.

The bottom number counts the equal parts. More parts means each part is smaller. So $^1/_8$ is smaller than $^1/_4$.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Which bar has $^1/_4$ shaded?



A



B

A Bar A

B Bar B

Answer at the back of the book.

2. A bread is cut into 5 equal parts. What is one part called?

   A one fifth

   B one fourth

   C one third

   Answer at the back of the book.

3. Which is the smaller part, one quarter or one eighth?

   A one quarter

   B one eighth

   Answer at the back of the book.

4. **From an earlier chapter**

   Mei cuts a flat bread into 4 pieces. The pieces are not the same size. Is one piece a quarter?

   A Yes. There are 4 pieces.

   B No. The parts must be equal.

   Answer at the back of the book.

5. Draw a bar. Cut it into 3 equal parts. Shade one part. What is the shaded part called, and how do you write it?

   Answer at the back of the book.

6. Ravi cuts a bread into 5 equal parts and takes one. How does he write his part?

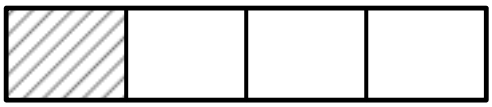
   A $^{1}/_{4}$

   B $^{5}/_{1}$

   C $^{1}/_{5}$

   Answer at the back of the book.

7. Mei shades one part of the bar. How do you write the shaded part?



a bar of bread

A $^1/_4$

B $^1/_3$

C $^1/_5$

Answer at the back of the book.

8. Three breads are the same size. Which part is smallest: $^1/_2$, $^1/_4$ or $^1/_8$?

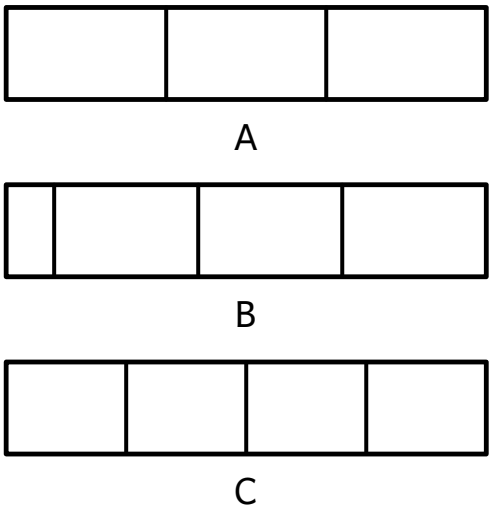
A $^1/_2$

B $^1/_8$

C They are all the same size.

Answer at the back of the book.

9. Which bar is cut into quarters?



A Bar A

B Bar B

C Bar C

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

The bottom number says how many equal parts. One part of 4 is $^1/_4$. More parts means smaller parts, so $^1/_8$ is smaller than $^1/_4$.

Draw: Two strips of the same length. Cut one into 4 equal parts and one into 8. Shade one part in each.

M2.4.3 CHAPTER 3

More than one part

If $^1/_4$ is one part, what do I call three parts?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** It is 1/3, because there are 3 parts.
- ☐ **B** It is 3/4, three of the quarter parts.
- ☐ **C** It is still 1/4, but bigger.

BY THE END YOU CAN

- Read and write a fraction such as $^3/_4$
- Say what the top and bottom numbers tell you
- Write a whole as $^4/_4$ or $^5/_5$

A **fraction** names some equal parts of a whole. Last time you named one part. Now you can take more.

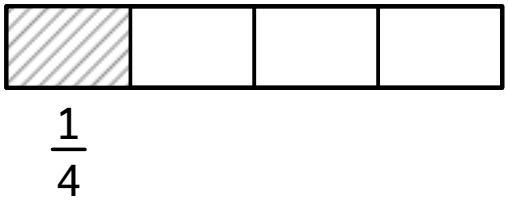
Cut a strip into 4 equal parts. Each part is $^1/_4$. Count the parts as you take them.

DRAW IT FIRST

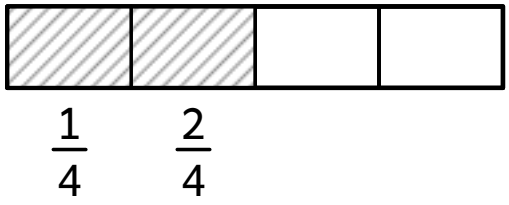
Count quarters to make three quarters



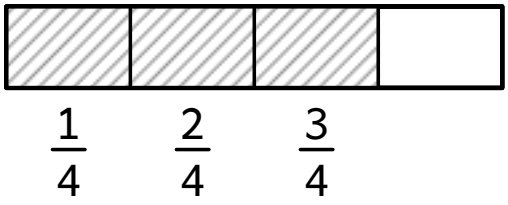
1 Draw a strip. Cut it into 4 equal parts.



2 Shade one part. That is $^1/_4$.



3 Shade a second part. Now 2 parts are shaded. That is $^2/_4$.



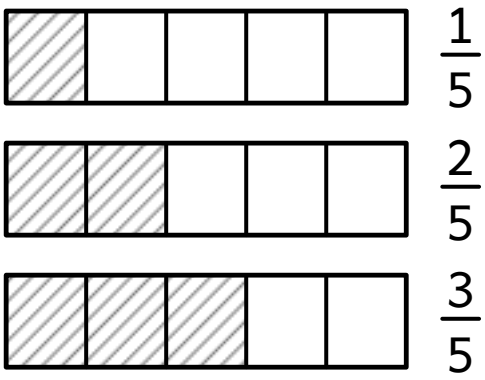
4 Shade a third part. Now 3 parts are shaded. That is $^3/_4$.

You counted 1, 2, 3 quarters. So the fraction is $^3/_4$. We say three quarters.

The bottom number is 4. It tells how many equal parts make the whole. It is also called the **denominator**.

The top number is 3. It tells how many parts you take. It is called the **numerator**.

Fifths work the same way. Cut a bar into 5 equal parts. Count the fifths you take.



Each bar has 5 equal parts. The top number counts the shaded parts.

What if you take every part? You have the whole. All 4 quarters make $^4/_4$. All 5 fifths make $^5/_5$.



four quarters, one whole



five fifths, one whole

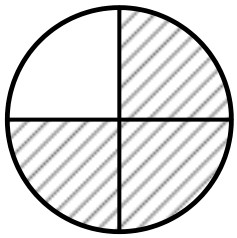
When the top number equals the bottom number, you have one whole.

SEE IT THREE WAYS

With things

Fold a paper into 4 equal parts. Open it. Colour 3 of the parts.

As a picture



three quarters

In symbols

3 parts of 4 equal parts is $^3/_4$.

WORKED EXAMPLE

A bar has 5 equal parts. Mei shades 2 parts. What fraction is shaded?

1. Count all the equal parts. There are 5. That is the bottom number.
2. Count the shaded parts. There are 2. That is the top number.
3. Write the top number over the bottom number.

Answer: $^{2}/_{5}$

TRY IT

Fold and colour three quarters

You need: a sheet of paper, or an old page, a pencil.

1. Fold the paper in half. Fold it in half again.
2. Open it. Count the equal parts. There are 4.
3. Colour 3 of the parts.
4. Count the coloured parts, then count all the parts.

What you will see: 3 coloured parts out of 4 equal parts is $^{3}/_{4}$. The part left over is $^{1}/_{4}$. Together they make $^{4}/_{4}$, the whole.

If you cannot try it, this is what happens.



three quarters

A WRONG IDEA MANY PEOPLE HAVE

~~The bottom number says how many parts you take.~~

Why it feels right: In $^{3}/_{4}$, the 4 is the bigger number. It looks like the count.

The test that breaks it: Fold a paper into 4 equal parts. Colour 3. Count the parts you coloured. Then count all the parts.

What is true: You coloured 3, and the paper has 4 parts. The top number counts the parts you take. The bottom number counts all the equal parts.

KEY WORDS

fraction
a number that names equal parts of a whole, like $^{3}/_{4}$.

numerator
the top number. It tells how many parts you take.

denominator
the bottom number. It tells how many equal parts make the whole.

BACK TO YOUR GUESS

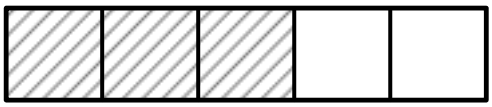
The answer is B: It is 3/4, three of the quarter parts.

Count the quarter parts you take. Three quarters is $^{3}/_{4}$. The top number counts the parts you take.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. What fraction of the bar is shaded?



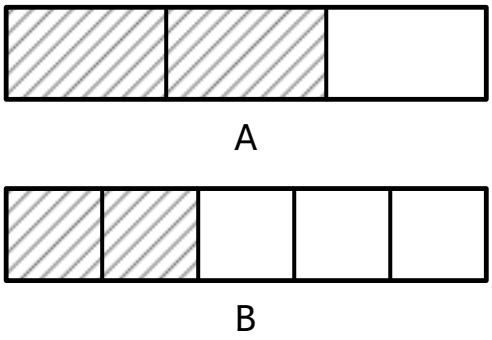
A $^{3}/_{5}$

B $^{2}/_{5}$

C $^{3}/_{2}$

Answer at the back of the book.

2. Which bar shows $^{2}/_{3}$?



A Bar A

B Bar B

Answer at the back of the book.

3. A bread is cut into 4 equal parts. Kofi eats all 4 parts. What fraction did he eat?

A $^{4}/_{4}$, the whole bread

B $^{1}/_{4}$

C $^{4}/_{1}$

Answer at the back of the book.

4. **From an earlier chapter**

   Which is the biggest part: one third, one quarter or one fifth?

   A one third

   B one quarter

   C one fifth

   Answer at the back of the book.

5. Draw a bar in 5 equal parts. Shade 3 parts. Write the fraction, and name the top and bottom numbers.

   Answer at the back of the book.

6. Sipho cuts a bread into 8 equal parts and eats 3. What does the 8 in $^{3}/_{8}$ tell you?

   A How many parts Sipho ate.

   B How many equal parts the whole bread has.

   C Which part Sipho ate.

   Answer at the back of the book.

7. Hana shades the whole bar. Which fraction names what she shaded?



a whole bar

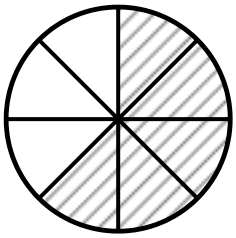
   A $^{1}/_{8}$

   B $^{8}/_{1}$

   C $^{8}/_{8}$

   Answer at the back of the book.

8. What fraction of the flat bread is shaded?



a flat bread

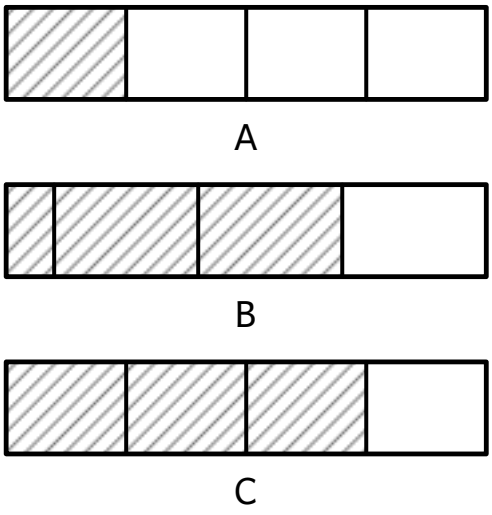
A $^{5}/_{3}$

B $^{5}/_{8}$

C $^{3}/_{8}$

Answer at the back of the book.

9. Which bar shows $^{3}/_{4}$?



A Bar A

B Bar B

C Bar C

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

The bottom number counts all the equal parts. The top number counts the parts you take. $^{3}/_{4}$ is three quarters. $^{4}/_{4}$ is one whole.

Draw: A strip in 4 equal parts. Shade 3 parts and write $^{3}/_{4}$ under it.

M2.4.4 CHAPTER 4

Fractions of *a* group

I have 12 beans. How many beans is $^1/_3$ of them?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 3 beans, because the bottom number is 3.
- ☐ **B** 4 beans.
- ☐ **C** 9 beans, because 12 take away 3.

BY THE END YOU CAN

- Find *a* fraction of *a* group of things
- Make equal groups, then take some of them
- Explain why $^2/_3$ of 12 is 8

So far the whole was one thing, like *a* bread. A whole can be *a* **group** of things too.

A group is *a* number of things together. 12 beans are *a* group. We can take *a* fraction of them.

Take $^1/_3$ of 12 beans. The bottom number is 3. Make 3 **equal groups**. Equal groups have the same number of things.

DRAW IT FIRST

Find one third of 12 beans

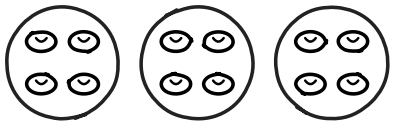


12 beans

1 Start with 12 beans. That is the whole group.



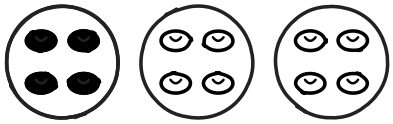
12 beans



2 Make 3 equal groups. Each group has 4 beans.



12 beans



one third of 12 is 4

3 Take 1 group. $^{1}/_{3}$ of 12 is 4.

The bottom number makes the groups. The top number says how many groups to take.

For $^{2}/_{3}$ of 12, make 3 equal groups. Then take 2 groups.

WORKED EXAMPLE

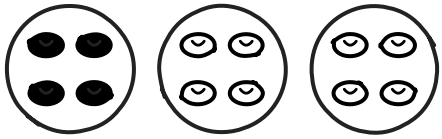
Find $^{2}/_{3}$ of 12 beans.

1. The bottom number is 3. Make 3 equal groups.
2. 12 beans in 3 equal groups is 4 in each group.
3. The top number is 2. Take 2 groups.
4. 2 groups of 4 is 4 + 4 = 8.

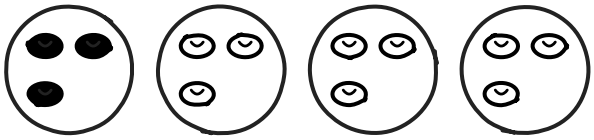
Answer: 8 beans

The same 12 beans can make 4 equal groups. Then each group is smaller. It holds fewer beans.

one third of 12 is 4



one quarter of 12 is 3



More groups means fewer beans in each group.

SEE IT THREE WAYS

With things	**As a picture**	**In symbols**
Put 8 stones in a pile. Share them into 4 equal groups. Take 3 groups.		**8 ÷ 4 = 2. 2 + 2 + 2 = 6. So $^{3}/_{4}$ of 8 is 6.**

TRY IT

Share beans into groups

You need: 12 beans, or 12 small stones, a flat table.

1. Count out 12 beans.
2. Share them one at a time into 3 groups. Stop when none are left.
3. Count one group.
4. Now push 2 groups together. Count them.

What you will see: Each group has 4. One group is $^1/_3$ of 12, which is 4. Two groups are $^2/_3$ of 12, which is 8.

If you cannot try it, this is what happens.



4 4 4

A WRONG IDEA MANY PEOPLE HAVE

~~$^1/_3$ of 12 is 3, because of the 3.~~

Why it feels right: The 3 is the only number you see in the fraction. It looks like the answer.

The test that breaks it: Count out 12 beans. Make 3 equal groups. Count the beans in one group.

What is true: The 3 says how many groups to make. Each group has 4 beans, so $^1/_3$ of 12 is 4.

KEY WORDS

group
a number of things together.

equal groups

groups that all have the same number of things.

BACK TO YOUR GUESS

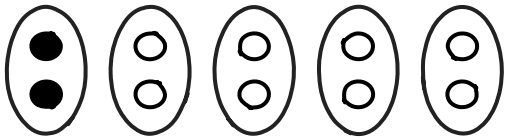
The answer is B: 4 beans.

Make 3 equal groups of the 12 beans. Each group has 4 beans. So $^1/_3$ of 12 is 4.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Look at the picture. How many stones are in $^1/_5$ of 10 stones?



   A 2 stones

   B 5 stones

   C 8 stones

   Answer at the back of the book.

2. What is $^1/_4$ of 12 beans?

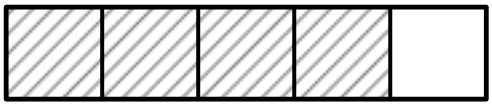
   Answer at the back of the book.

3. What is $^2/_3$ of 9 stones?

   Answer at the back of the book.

4. **From an earlier chapter**

   A bar has 5 equal parts. 4 parts are shaded. What fraction is shaded?



   A $^4/_5$

   B $^1/_5$

   C $^5/_4$

   Answer at the back of the book.

5. Mei has 12 beans. She gives $^3/_4$ of them to Kofi. How many beans does Kofi get? How many does Mei keep?

   Answer at the back of the book.

6. Amina finds $^2/_3$ of 12 beans. Which working is right?

   A Divide 12 by 2. That makes 6 beans.

   B Make 3 equal groups of 4. Take 2 groups. That makes 8 beans.

   C Make 3 equal groups of 4. Take 1 group. That makes 4 beans.

   Answer at the back of the book.

7. Hana has 10 beans, as in the picture. What is $^3/_5$ of them?



   10 beans

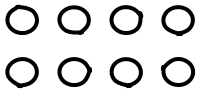
   A 6 beans

   B 2 beans

   C 5 beans

   Answer at the back of the book.

8. Tariq finds $^3/_4$ of 8 stones. How many equal groups does he make first?



8 stones

A 4 groups

B 3 groups

C 2 groups

Answer at the back of the book.

9. Priya has 6 stones in equal groups, as in the picture. She takes 1 group. What fraction of the stones is that?



2 stones 2 stones 2 stones

A $^1/_2$

B $^1/_3$

C $^1/_6$

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

The bottom number says how many equal groups to make. The top number says how many groups to take. $^2/_3$ of 12 is 8.

Draw: 12 dots in 3 groups of 4. Fill in 2 groups. Write 4 + 4 = 8.

M2.4.5 CHAPTER 5

The number line, past 1

I saw 5/4 on a page. Can a fraction be more than 1?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** No. A fraction is a piece, so it is always less than 1.
- ☐ **B** Yes. Count more parts than one whole has. You go past 1.
- ☐ **C** No. Only whole numbers can go past 1.

BY THE END YOU CAN

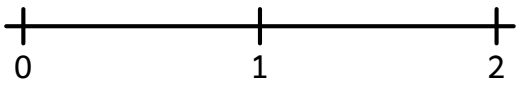
- Find a fraction on a number line
- Tell if a fraction is before 1, at 1, or past 1
- Count steps past 1 to place a fraction such as $^5/_4$

A **number line** is a straight line with numbers on it. Each number has its own place.

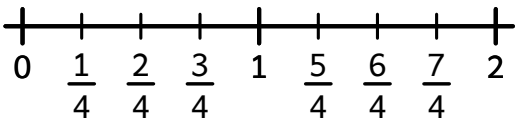
Fractions are numbers too. They have places on the line, between the whole numbers.

DRAW IT FIRST

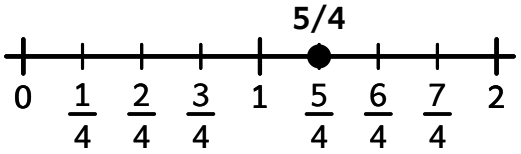
Find $^{5}/_{4}$ on *a* number line



1 Draw *a* line. Mark 0, 1 and 2 with equal gaps.

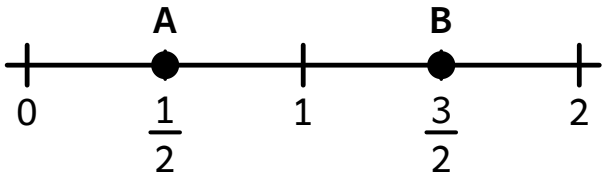


2 Cut each whole into 4 equal **steps**. Each step is $^{1}/_{4}$.



3 Count 5 steps from 0. You land on $^{5}/_{4}$. It is past 1.

Four steps of $^{1}/_{4}$ reach 1. So $^{4}/_{4}$ is the same place *as* 1.



A is $^{1}/_{2}$. It is before 1. B is $^{3}/_{2}$. It is past 1.

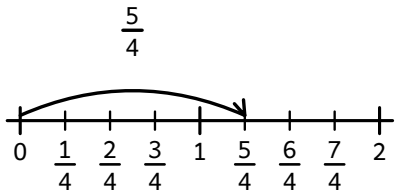
Look *at* the top *and* bottom numbers. Is the top bigger than the bottom? Then the fraction is past 1.

SEE IT THREE WAYS

With things

Take two strings, each 1 metre long. Cut each one into 4 equal pieces. Lay 5 pieces end to end.

As a picture



In symbols

$5/4 = 4/4 + 1/4$. That is 1 and one more quarter.

TRY IT

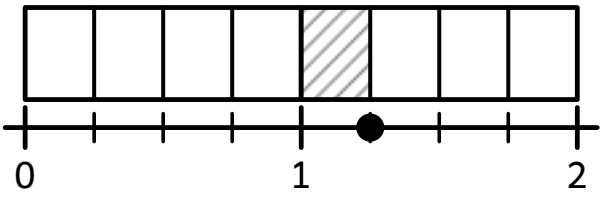
Walk past 1 with paper strips

You need: two strips of paper of the same length, a pencil.

1. Fold one strip in half. Fold it in half again. Open it.
2. Do the same with the other strip. Each strip has 4 equal parts.
3. Lay the strips end to end. Write 0 at the left end.
4. Count 4 parts from 0. Where the first strip ends, write 1.
5. Count one more part. Write $5/4$ at the end of that part.

What you will see: The fifth part is on the second strip. So $5/4$ is past 1.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~A fraction is always less than 1.~~

Why it feels right: Most fractions we cut are pieces of one whole.

The test that breaks it: Cut two breads of the same size, each into 4 equal pieces. Take 5 pieces. You hold more than one whole bread.

What is true: A fraction can be less than 1. It can equal 1. It can be more than 1. If the top is bigger than the bottom, it is past 1.

KEY WORDS

number line
a line where every number has its own place.

steps
the equal parts of a whole, on a number line.

BACK TO YOUR GUESS

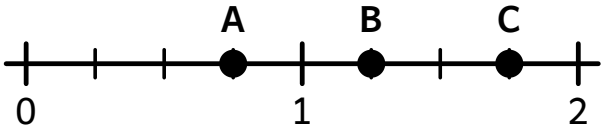
The answer is B: Yes. Count more parts than one whole has. You go past 1.

A fraction is a number. It has a place on the number line. Five quarters sits after 1.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Count the steps from 0. Which point is at $^{5}/_{4}$?



A Point A

B Point B

C Point C

Answer at the back of the book.

2. Is $^{6}/_{4}$ before 1, at 1, or past 1?

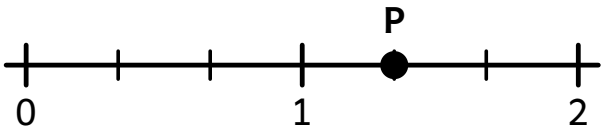
A Before 1

B At 1

C Past 1

Answer at the back of the book.

3. Each whole is cut into 3 equal steps. Point P is the first tick after 1. What fraction is P?



Answer at the back of the book.

4. **From an earlier chapter**

Amina eats 3 of the 4 equal parts of a bread. What fraction of the bread did she eat?

A 3/4

B 1/4

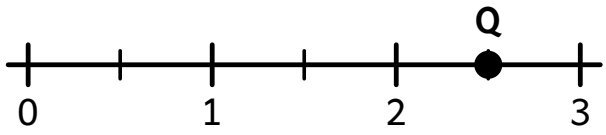
C 4/3

Answer at the back of the book.

5. Draw a number line from 0 to 2. Cut each whole in half. Mark $^{3}/_{2}$.

   Answer at the back of the book.

6. What fraction is point Q?



   A $^{2}/_{5}$

   B $^{5}/_{2}$

   C $^{6}/_{2}$

   Answer at the back of the book.

7. Which of these fractions is past 1?

   A $^{5}/_{8}$

   B $^{6}/_{6}$

   C $^{9}/_{6}$

   Answer at the back of the book.

8. Kofi counts steps of $^{1}/_{3}$ from 0 and lands on $^{5}/_{3}$. How many steps does he count?

   A 5 steps

   B 3 steps

   C 2 steps

   Answer at the back of the book.

9. Priya says $^{7}/_{4}$ is past 1. She says the top is bigger than the bottom. Is she right?

   A No. A fraction is always less than 1.

   B Yes. The top is bigger than the bottom.

   C No. It is at 1. 7 parts make one whole.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A fraction is a number with a place on the number line. Cut each whole into equal steps. Count the steps from 0. Top bigger than bottom? You are past 1.

Draw: A line from 0 to 2. Cut each whole into 4 steps. Mark 5/4, just past 1.

M2.4.6 CHAPTER 6

Same amount, different names

Is 2/4 more than 1/2, since the numbers are bigger?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Yes. 2 and 4 are bigger than 1 and 2.
- ☐ **B** No. They are the same amount.
- ☐ **C** No. 2/4 is less than 1/2.

BY THE END YOU CAN

- Show that two fractions name the same amount
- Find a fraction equal to $^1/_2$, $^1/_3$ or $^3/_4$
- Explain why $^2/_4$ is not more than $^1/_2$

Cut a bread into 2 equal halves. Then cut each half into 2 equal pieces. Now you have 4 pieces in all.

You still have the same bread. It has the same size. Only the cuts are new. One half is now 2 of the 4 pieces.

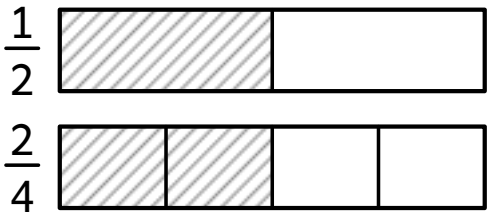
Fractions that name the same amount are **equal fractions**. We write $^1/_2 = {}^2/_4$.

DRAW IT FIRST

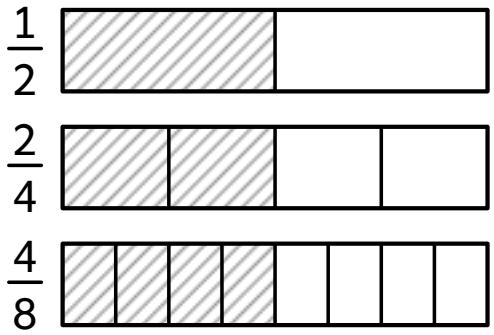
Cut each part in two



1 Draw a bar. Cut it in 2 equal parts. Shade 1 part. That is $^1/_2$.

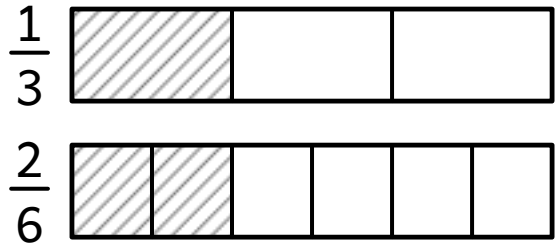


2 Draw a bar the same length. Cut it in 4. Shade 2 parts. That is $^2/_4$.

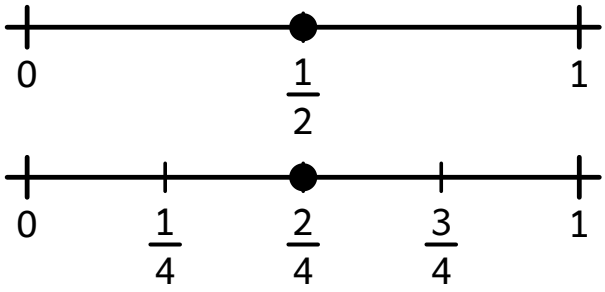


3 Draw one more bar the same length. Cut it in 8. Shade 4 parts. That is $^4/_8$.

Bars with the same length have the **same whole**. Look down the bars. Each shaded part ends in the same place.



$^1/_3 = {}^2/_6$. Cut each third in 2, and 1 part becomes 2 parts.



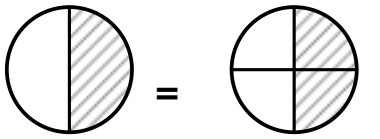
On equal lines, $^1/_2$ and $^2/_4$ are in the same place.

SEE IT THREE WAYS

With things

Take two breads of the same size. Cut one in 2 and one in 4. Put 1 piece of the first by 2 pieces of the second.

As a picture



In symbols

$^1/_2 = {}^2/_4 = {}^4/_8$. Each time, the top and the bottom both double.

WORKED EXAMPLE

Make a fraction equal to $^2/_3$ that has 6 parts.

1. Cut each third in 2. The 3 parts become 6 parts.
2. Each shaded part is cut in 2 too. The 2 shaded parts become 4.
3. The bottom doubled, from 3 to 6. The top doubled, from 2 to 4.

Answer: $^2/_3 = {}^4/_6$

TRY IT

Two strips, one amount

You need: two strips of paper of the same length, a pencil.

1. Fold the first strip in half. Open it. Shade one half.
2. Fold the second strip in half, then in half again. Open it.
3. Shade 2 of the 4 parts on the second strip.
4. Put the strips side by side, with the left ends together.

What you will see: Both shaded parts end in the same place. So $^1/_2$ and $^2/_4$ are the same amount.

If you cannot try it, this is what happens.



one half



two quarters

A WRONG IDEA MANY PEOPLE HAVE

~~**$^2/_4$ is more than $^1/_2$, because 2 and 4 are bigger numbers.**~~

Why it feels right: With whole numbers, a bigger number means more.

The test that breaks it: Make the two strips from the Try it. Lay them side by side. The shaded parts end in the same place. Neither sticks out.

What is true: Cutting parts smaller gives more parts. The amount does not change. $^2/_4$ is the same amount as $^1/_2$.

KEY WORDS

equal fractions
fractions that name the same amount.

same whole
bars or lines that have the same length.

BACK TO YOUR GUESS

The answer is B: No. They are the same amount.

Two quarters is the same amount as one half. Bigger numbers do not mean more. The parts are smaller, and there are more of them.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Which fraction is equal to $^1/_2$?

   A 2/4

   B 1/4

   C 2/3

   Answer at the back of the book.

2. This bar has 3 equal parts, with 1 shaded. Cut each part in 2. How many parts are shaded now?



before

   Answer at the back of the book.

3. Is $^3/_6$ equal to $^1/_2$?

   A Yes. Both name the same amount.

   B No. 3/6 is more, because 3 and 6 are big.

   Answer at the back of the book.

4. **From an earlier chapter**

   A bread is cut into 4 equal parts. Mei eats one part. What fraction of the bread is that?

   A 1/4

   B 1/3

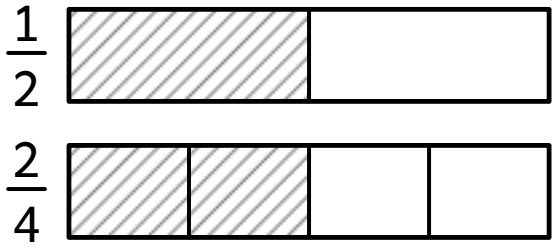
   C 4/1

   Answer at the back of the book.

5. Fill the gap. 3/4 = ?/8. What number goes in place of ?

   Answer at the back of the book.

6. Look at the bars. Is $^{2}/_{4}$ more than $^{1}/_{2}$?



   A Yes. It has more parts shaded.

   B No. They are equal. The shaded parts have the same length.

   C No. It is less, because the parts are smaller.

   Answer at the back of the book.

7. Which fraction is equal to $^{1}/_{3}$?

   A $^{1}/_{6}$

   B $^{2}/_{4}$

   C $^{2}/_{6}$

   Answer at the back of the book.

8. This bar has 4 equal parts, with 3 shaded. Cut each part in 2. What fraction is shaded now?



before

A $^{6}/_{8}$

B $^{3}/_{8}$

C $^{4}/_{5}$

Answer at the back of the book.

9. Which pair of fractions names the same amount?

A $^{1}/_{4}$ and $^{1}/_{8}$

B $^{2}/_{6}$ and $^{1}/_{3}$

C $^{2}/_{3}$ and $^{3}/_{4}$

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Different fractions can name the same amount. 1/2, 2/4 and 4/8 are equal. Cut every part in two. The top and the bottom both double. The amount stays the same.

Draw: Three bars of the same length, cut in 2, 4 and 8. Shade the same amount in each.

M2.4.7 CHAPTER 7

Which is bigger

Which is more, 1/4 or 1/8, since 8 is bigger than 4?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 1/8, because 8 is bigger than 4.
- ☐ **B** 1/4, because its parts are bigger.
- ☐ **C** They are the same amount.

BY THE END YOU CAN

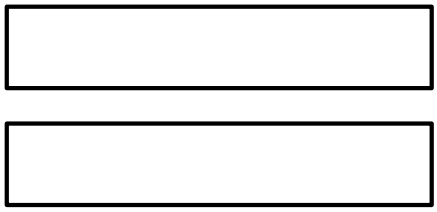
- Compare two fractions with the same bottom number
- Compare two fractions with the same top number
- Explain why $^1/_8$ is less than $^1/_4$

To **compare** two fractions is to find which one is more. Or to find that they are the same.

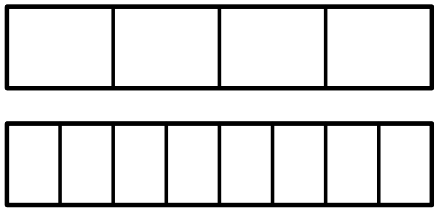
Always draw bars with the **same whole**. Then the bars have the same length, and the picture is fair.

DRAW IT FIRST

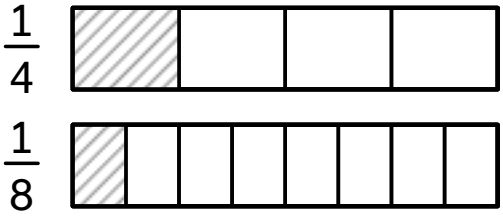
Compare $^1/_4$ and $^1/_8$



1 Draw two bars of the same length. Each bar is one whole.



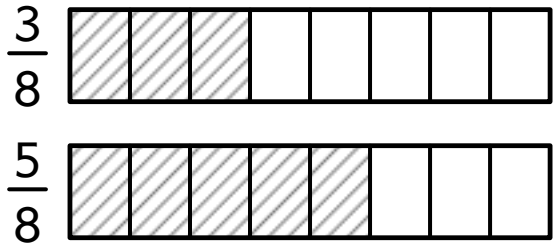
2 Cut one bar in 4 equal parts. Cut the other in 8 equal parts.



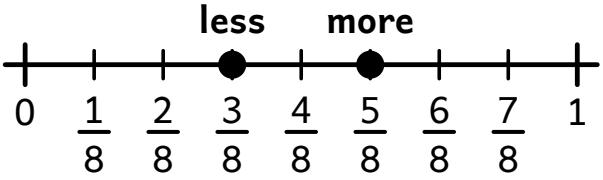
3 Shade one part in each bar. The $^1/_8$ part is shorter than the $^1/_4$ part.

So $^1/_8$ is less than $^1/_4$. More parts in the whole means smaller parts.

First rule. The same bottom number means parts of the same size. More parts is more.

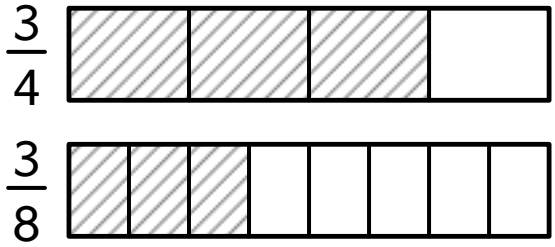


The parts are the same size. 5 parts are more than 3 parts.



On the number line, further right means more.

Second rule. When the top numbers are the same, look at the bottom numbers.



Both show 3 parts. The parts of $^{3}/_{4}$ are bigger.

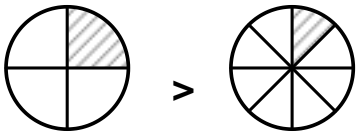
A bigger bottom number means smaller parts. So $^{3}/_{8}$ is less than $^{3}/_{4}$.

SEE IT THREE WAYS

With things

Take two breads of the same size. Cut one in 4 and one in 8. Take 1 piece from each.

As a picture



one quarter one eighth

In symbols

$^{1}/_{4} > {}^{1}/_{8}$. The open side of the sign faces the bigger one.

TRY IT

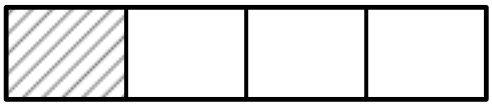
Fold two strips

You need: two strips of paper of the same length, a pencil.

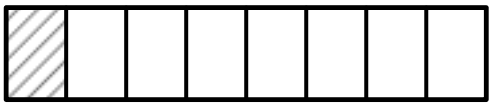
1. Fold the first strip in half. Fold it in half again. Open it. It has 4 parts.
2. Fold the second strip in half 3 times. Open it. It has 8 parts.
3. Shade one part on each strip.
4. Put the strips side by side, with the left ends together.

What you will see: The shaded part of the second strip is half as long. So $^1/_8$ is less than $^1/_4$.

If you cannot try it, this is what happens.



one quarter



one eighth

A WRONG IDEA MANY PEOPLE HAVE

~~**$^1/_8$ is more than $^1/_4$, because 8 is bigger than 4.**~~

Why it feels right: With whole numbers, 8 is more than 4. The same rule seems to work here.

The test that breaks it: Share one bread between 4 people. Share another bread of the same size between 8 people. Ask who gets the bigger piece.

What is true: The bottom number is how many parts the whole is cut into. More parts means smaller parts. So $^1/_8$ is less than $^1/_4$.

KEY WORDS

compare

to find which is more, or that they are the same.

same whole
bars or lines that have the same length.

BACK TO YOUR GUESS

The answer is B: 1/4, because its parts are bigger.

The bottom number tells how many parts the whole is cut into. More parts means smaller parts. So 1/4 is more than 1/8.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Which is more, $^{3}/_{8}$ or $^{5}/_{8}$?

   A 3/8

   B 5/8

   Answer at the back of the book.

2. Which is less, $^{1}/_{3}$ or $^{1}/_{6}$?

   A 1/3

   B 1/6

   Answer at the back of the book.

3. **From an earlier chapter**

   Which fraction is equal to $^{1}/_{3}$?

   A 2/6

   B 1/6

   C 2/3

   Answer at the back of the book.

4. Put these in order, from least to most: $^1/_2$, $^1/_8$, $^1/_4$.

   Answer at the back of the book.

5. Kofi eats $^3/_4$ of a bread. Lucía eats $^3/_8$ of a bread of the same size. Who ate more?

   A Kofi

   B Lucía

   C They ate the same

   Answer at the back of the book.

6. Ravi has $^5/_8$ of a bread. Amina has $^3/_8$ of a bread of the same size. Who has more?

   A Ravi

   B Amina

   C They have the same amount.

   Answer at the back of the book.

7. Which is more, $^3/_5$ or $^3/_{10}$?

   A $^3/_{10}$

   B They are the same.

   C $^3/_5$

   Answer at the back of the book.

8. Mei says $^1/_8$ is more than $^1/_4$. Her reason is that 8 is more than 4. Is she right?

   A Yes. A bigger number is always more.

   B No. More parts in the whole means smaller parts.

   C No. They are the same, because both have 1 on top.

   Answer at the back of the book.

9. Put $^3/_4$, $^3/_5$ and $^3/_8$ in order, least to most. Which list is right?

   A $^3/_8$, $^3/_5$, $^3/_4$

   B $^3/_4$, $^3/_5$, $^3/_8$

   C They are all the same amount.

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Compare fractions on bars of the same whole. Same bottom number: more parts is more. Same top number: a bigger bottom number means smaller parts. So 1/8 is less than 1/4.

Draw: Two bars of the same length. Cut one in 4 and one in 8. Shade one part in each.

M2.4.8 CHAPTER 8

Adding and taking away parts

My friend says 1/4 + 1/4 is 2/8. Is that right?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 2/8. Add the tops and add the bottoms.
- ☐ **B** 2/4. Add the tops and keep the bottom.
- ☐ **C** You cannot add fractions.

BY THE END YOU CAN

- Add fractions that have the same bottom number
- Take away fractions that have the same bottom number
- Show adding and taking away as hops on a number line

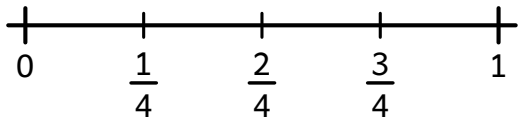
Two quarters of a bread make half the bread. Let us see why.

In this chapter the bottom numbers are always the same. Parts with the same bottom number are **like parts**.

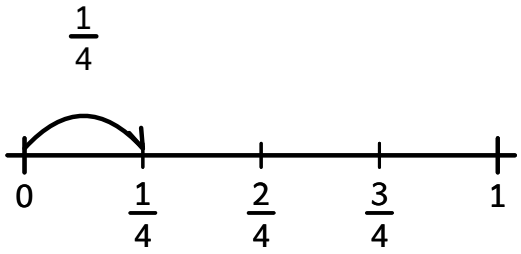
You can count like parts as you count beans. 1 quarter and 2 quarters make 3 quarters.

DRAW IT FIRST

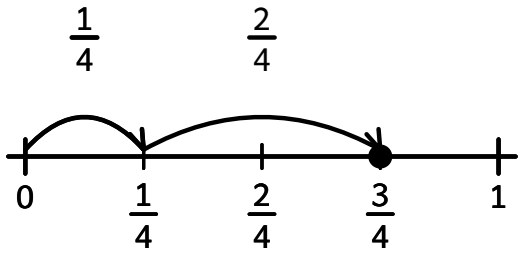
Add $^1/_4$ and $^2/_4$ with hops



1 Draw a number line from 0 to 1. Cut it into quarters.



2 Start at 0. Make a **hop** of $^1/_4$ to the right.



3 Hop $^2/_4$ more. You land on $^3/_4$. So $^1/_4 + ^2/_4 = ^3/_4$.

The hops are all quarters. So the bottom number stays 4. Only the tops are added.

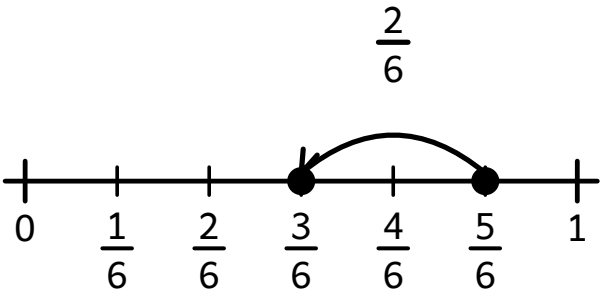
WORKED EXAMPLE

Add $^1/_4 + ^2/_4$.

1. The bottom numbers are both 4. The parts are the same size.
2. Add the top numbers. 1 + 2 = 3.
3. Keep the bottom number. It is still 4.

Answer: $^3/_4$

To take away, hop to the left. Start at $^5/_6$ and take away $^2/_6$.



Hop left by $^{2}/_{6}$. You land on $^{3}/_{6}$.

WORKED EXAMPLE

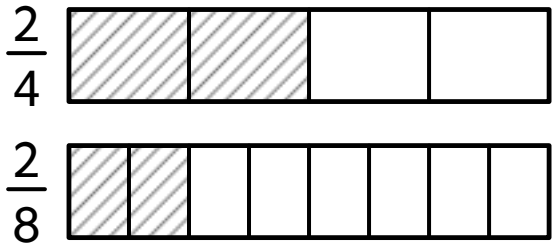
Take away $^{5}/_{6} - {}^{2}/_{6}$.

1. The bottom numbers are both 6.
2. Take away the top numbers. 5 − 2 = 3.
3. Keep the bottom number. It is still 6.

Answer: $^{3}/_{6}$. That is the same amount as $^{1}/_{2}$.

Sometimes you hop past 1. Then the top is bigger than the bottom. For example, $^{3}/_{4} + {}^{2}/_{4} = {}^{5}/_{4}$.

Now back to the question. Is $^{1}/_{4} + {}^{1}/_{4}$ the same as $^{2}/_{8}$? Look at the bars.



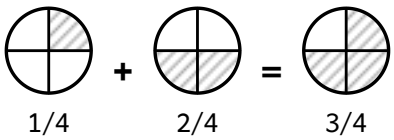
$^{1}/_{4} + {}^{1}/_{4} = {}^{2}/_{4}$. Two quarters cover half the bar. Two eighths cover only a quarter.

SEE IT THREE WAYS

With things

Cut *a* bread in 4 equal pieces. Eat 1 piece. Then eat 2 more. You ate 3 pieces of 4.

As *a* picture



In symbols

$^1/_4 + {}^2/_4 = {}^3/_4$. Add the tops. Keep the bottom.

TRY IT

Add on *a* paper strip

You need: *a* strip of paper, *a* pencil.

1. Fold the strip in half. Fold it in half again. Open it. It has 4 equal parts.
2. Shade 1 part.
3. Shade 2 more parts, next to the first.
4. Count the shaded parts.

What you will see: You shaded 3 of the 4 parts. So $^1/_4 + {}^2/_4 = {}^3/_4$. Every part is *a* quarter, so the bottom stays 4.

If you cannot try it, this is what happens.



three quarters

A WRONG IDEA MANY PEOPLE HAVE

~~$^1/_4 + {}^1/_4 = {}^2/_8$.~~

Why it feels right: You add the tops, 1 + 1. Then you add the bottoms, 4 + 4. Adding everything seems fair.

The test that breaks it: Cut a bread in 4 equal parts. Eat 1 part, then 1 more. You ate half the bread. Now cut a bread in 8. Two of those pieces are only a quarter.

What is true: The parts stay quarters, so the bottom stays 4. $^1/_4 + {}^1/_4 = {}^2/_4$. That is $^1/_2$.

KEY WORDS

like parts
parts that have the same bottom number.

hop
a jump along the number line.

BACK TO YOUR GUESS

The answer is B: 2/4. Add the tops and keep the bottom.

Two quarters of a bread are half the bread. That is 2/4. And 2/8 is only a quarter. Add the tops and keep the bottom.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Add. $^2/_7 + {}^3/_7$

   A 5/7

   B 5/14

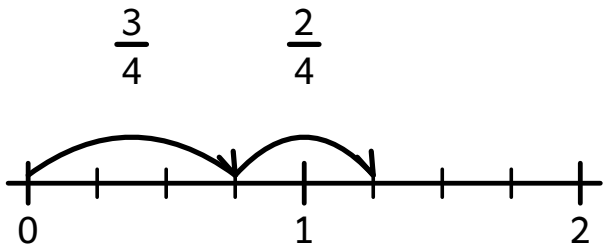
   C 6/7

Answer at the back of the book.

2. Take away. $^5/_6 - {}^2/_6$

   Answer at the back of the book.

3. Ravi hops along this line. He hops $^3/_4$, then $^2/_4$. Where does he land?



   Answer at the back of the book.

4. **From an earlier chapter**

   What is $^1/_3$ of 12 stones?

   Answer at the back of the book.

5. A jug has $^7/_8$ litre of water. Ana pours out $^3/_8$ litre. How much is left?

   Answer at the back of the book.

6. Leila eats $^1/_6$ of a bread. Then she eats $^3/_6$ more. How much has she eaten in all?

   A $^4/_{12}$

   B $^4/_6$

   C $^3/_6$

   Answer at the back of the book.

7. Yusuf has $^5/_8$ of a bread. He gives $^2/_8$ away. How much is left?

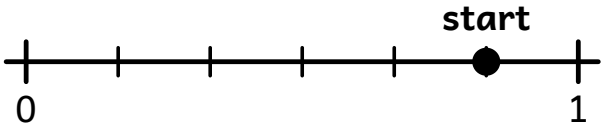
   A $^7/_8$

   B $^2/_8$

   C $^3/_8$

   Answer at the back of the book.

8. Hana starts at $^5/_6$ and hops left by $^3/_6$. Where does she land?



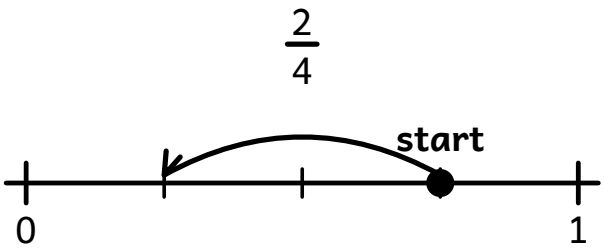
A $^2/_6$

B $^8/_6$

C $^3/_6$

Answer at the back of the book.

9. Mei hops to the left on the line. Which number sentence shows her hop?



A $^3/_4 + {}^2/_4 = {}^5/_4$

B $^3/_4 - {}^2/_4 = {}^1/_4$

C $^3/_4 - {}^2/_4 = {}^2/_4$

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Same bottom number: add or take away the tops. Keep the bottom. 1/4 + 2/4 = 3/4. Two quarters of a bread make half the bread. 1/4 + 1/4 is 2/4, never 2/8.

Draw: A number line in quarters. Hop 1/4, then hop 2/4, and land on 3/4.

Key words, A to Z

Every key word in this book, what it means, and the chapter that teaches it.

bottom number
the number under the line. It tells how many equal parts the whole is cut into. M2.4.2

compare
to find which is more, or that they are the same. M2.4.7

denominator
the bottom number. It tells how many equal parts make the whole. M2.4.3

equal fractions
fractions that name the same amount. M2.4.6

equal groups
groups that all have the same number of things. M2.4.4

equal parts
parts that are all the same size. M2.4.1

fifth
one of 5 equal parts of a whole. It is written $^1/_5$. M2.4.2

fraction
a number that names equal parts of a whole, like $^3/_4$. M2.4.3

group
a number of things together. M2.4.4

half
one of two equal parts of a whole. M2.4.1

hop
a jump along the number line. M2.4.8

like parts
parts that have the same bottom number. M2.4.8

number line
a line where every number has its own place. M2.4.5

numerator
the top number. It tells how many parts you take. M2.4.3

quarter

one of 4 equal parts of a whole. It is written $^1/_4$. Also called a fourth. M2.4.2

same whole
bars or lines that have the same length. M2.4.6, M2.4.7

steps
the equal parts of a whole, on a number line. M2.4.5

third
one of 3 equal parts of a whole. It is written $^1/_3$. M2.4.2

whole
all of one thing, before it is cut. M2.4.1

Answers

M2.4.1. Fair shares: equal parts

1. **Bar B**

   B has two parts of the same size. A has a small part and a big part.

   - **If you chose A:** You may be counting pieces. Bar A has two pieces, but they are not the same size.

2. **4 stones each**

   12 stones in 3 equal groups is 4 in each group. 4 + 4 + 4 = 12.

3. **No. A fair share is 5 each.**

   10 beans shared by 2 is 5 each. 6 and 4 are not the same.

   - **If you chose A:** Both having some is not enough. A fair share means the same number each.

4. **One way: a line across and a line down, through the middle. Another way: three lines from top to bottom. They make 4 strips of the same width.**

   Both ways give 4 parts that are all the same size. There are many more ways.

5. **Bar B**

   Bar B has 3 parts of the same size. In A and C some parts are bigger than others.

   - **If you chose A:** Bar A has 3 parts, but all three are different sizes. Counting the parts is not enough.
   - **If you chose C:** Two parts match. But the first part is twice as long as one of them. All 3 must match.

6. **3 beans each**

   12 beans in 4 equal groups is 3 in each group. 3 + 3 + 3 + 3 = 12.

   - **If you chose A:** There are 4 friends, but that is not the share. Make 4 equal groups of the 12 beans. Each group has 3.
   - **If you chose B:** You took 4 away from 12. Sharing between 4 friends means making 4 equal groups.

7. **Give 5 beans to each child.**

   15 beans in 3 equal groups is 5 in each group. 5 + 5 + 5 = 15.

   - **If you chose B:** Every child gets some, and 7 + 5 + 3 = 15. But a fair share gives every child the same number.
   - **If you chose C:** 3 + 3 + 3 = 9, so 6 beans are left over. A fair share of 15 beans uses all of them.

8. **Lay one piece on top of another. They match exactly.**

   Pieces that match exactly are the same size.

   - **If you chose A:** Counting tells how many pieces there are, not how big.
   - **If you chose B:** Straight cuts can still leave one big piece and one small piece.

9. **No. The two parts are not the same size.**

   One part is big and one part is small. Halves are two parts of the same size.

   - **If you chose A:** Two parts are not enough. Halves must be the same size.
   - **If you chose C:** Two parts of any size can make a whole bread. Halves must also be the same size.

M2.4.2. One part: halves, quarters, thirds, fifths

1. **Bar A**

   Bar A has 4 equal parts and one is shaded. That is $^1/_4$.

   - **If you chose B:** Count the equal parts. Bar B has 3 parts, so one part is a third, $^1/_3$.

2. **one fifth**

   The bottom number is the number of equal parts. 5 parts gives fifths.

   - **If you chose B:** A fourth is one of 4 equal parts. Count again. This bread has 5.
   - **If you chose C:** A third is one of 3 equal parts. This bread has 5.

3. **one eighth**

   Cutting a whole into 8 parts gives smaller parts than 4 parts.

   - **If you chose A:** 8 is more than 4, but that means more parts. More parts are smaller parts.

4. **No. The parts must be equal.**

   Quarters, halves and thirds are always equal parts. Pieces of different sizes are not.

   - **If you chose A:** Counting pieces is not enough. A quarter must be one of 4 equal parts. Go to M2.4.1.

5. **One third, written $^{1}/_{3}$.**

   The bar has 3 equal parts, so the bottom number is 3. You took one part, so the top is 1.

6. $^{1}/_{5}$

   The bread has 5 equal parts, so the bottom number is 5. Ravi takes 1 part, so the top is 1.

   - **If you chose A:** The bottom number counts all 5 equal parts. It does not count the parts left over.
   - **If you chose B:** The top number says how many parts Ravi takes, which is 1. The bottom counts all the equal parts.

7. $^{1}/_{4}$

   The bar has 4 equal parts, so the bottom number is 4. One part is shaded, so the top is 1.

   - **If you chose B:** You counted the parts left over, which is 3. The bottom number counts all the equal parts.
   - **If you chose C:** You counted the lines, which is 5. Count the parts between the lines.

8. $^{1}/_{8}$

   The bread cut into 8 parts has the most parts. So each part is the smallest.

   - **If you chose A:** 2 is the smallest bottom number. But fewer parts means each part is bigger.
   - **If you chose C:** Each one is 1 part. But the bottom number says how many parts. More parts means smaller parts.

9. **Bar C**

   Bar C has 4 parts of the same size. Bar B has 4 parts that are not equal.

   - **If you chose A:** Bar A has 3 equal parts. That makes thirds. Count the parts, not the lines.
   - **If you chose B:** Bar B has 4 parts, but they are not the same size. Quarters are equal parts. Go to M2.4.1.

M2.4.3. More than one part

1. **$^{3}/_{5}$**

   There are 5 equal parts, so the bottom number is 5. 3 are shaded, so the top number is 3.

   - **If you chose B:** You counted the parts that are not shaded. The top number counts the shaded parts.
   - **If you chose C:** You may have put shaded parts over parts not shaded. The bottom number counts all the equal parts. That is 5, not 2. Go to M2.4.2.

2. **Bar A**

   Bar A has 3 equal parts and 2 are shaded. That is $^{2}/_{3}$.

   - **If you chose B:** Bar B has 5 equal parts, so it shows $^{2}/_{5}$. The bottom number must be 3. Go to M2.4.2.

3. **$^{4}/_{4}$, the whole bread**

   The bread has 4 equal parts. He took 4 of them. So the top and bottom numbers are both 4.

   - **If you chose B:** That is one part. Kofi ate 4 parts.
   - **If you chose C:** The bottom number is the number of equal parts in the whole. That is 4. Go to M2.4.2.

4. **one third**

   Cutting into 3 parts gives bigger parts than 4 or 5.

   - **If you chose B:** Cutting into 3 parts gives bigger parts than 4. Go to M2.4.2.
   - **If you chose C:** More parts means smaller parts. Cutting into 5 gives the smallest part here. Go to M2.4.2.

5. **$^{3}/_{5}$. The top number is 3. The bottom number is 5.**

   The bar has 5 equal parts. You shaded 3 of them. The top number counts shaded parts.

6. **How many equal parts the whole bread has.**

   The bread is cut into 8 equal parts. The bottom number counts all of them.

   - **If you chose A:** Sipho ate 3 parts, the top number. The bottom number counts all the equal parts.
   - **If you chose C:** The bottom number counts the equal parts. It does not say which part he ate.

7. **$^{8}/_{8}$**

   The bar has 8 equal parts and Hana shaded all 8. The top and bottom numbers are both 8.

   - **If you chose A:** That is one part. Hana shaded every part.
   - **If you chose B:** The bottom number counts the equal parts in the whole. That is 8, not 1.

8. **$^{5}/_{8}$**

   There are 8 equal slices, so the bottom number is 8. 5 are shaded, so the top number is 5.

   - **If you chose A:** You put shaded parts over parts not shaded. The bottom number counts all the equal parts, 8.
   - **If you chose C:** You counted the parts that are not shaded. The top number counts the shaded parts.

9. **Bar C**

   Bar C has 4 equal parts and 3 are shaded. That is $^{3}/_{4}$.

   - **If you chose A:** You counted the parts that are not shaded. The top number counts the shaded parts.
   - **If you chose B:** Bar B has 3 parts shaded. But its 4 parts are not equal. A fraction needs equal parts. Go to M2.4.1.

M2.4.4. Fractions of *a* group

1. **2 stones**

   10 stones in 5 equal groups is 2 in each group. $^1/_5$ of 10 is one group.

   - **If you chose B:** The 5 tells how many groups to make. It does not tell how many stones.
   - **If you chose C:** You counted the stones not filled in. That is $^4/_5$ of 10. $^1/_5$ is 1 group.

2. **3 beans**

   12 beans in 4 equal groups is 3 in each group. One group is 3 beans.

3. **6 stones**

   9 stones in 3 equal groups is 3 in each group. Two groups is 3 + 3 = 6.

4. **$^4/_5$**

   The bar has 5 equal parts. 4 are shaded. So the fraction is $^4/_5$.

   - **If you chose B:** That is one part. Here 4 parts are shaded, so the top number is 4. Go to M2.4.3.
   - **If you chose C:** The bottom number is all the equal parts. That is 5. Go to M2.4.3.

5. **Kofi gets 9 beans. Mei keeps 3 beans.**

   12 beans in 4 equal groups is 3 in each group. Kofi takes 3 groups, which is 9. Mei keeps 1 group, which is 3.

6. **Make 3 equal groups of 4. Take 2 groups. That makes 8 beans.**

   12 beans in 3 equal groups is 4 in each group. 2 groups is 4 + 4 = 8.

   - **If you chose A:** The bottom number, 3, makes the groups. The top number says how many to take.
   - **If you chose C:** That is $^1/_3$ of 12. The top number is 2, so take 2 groups.

7. **6 beans**

   10 beans in 5 equal groups is 2 in each group. 3 groups is 2 + 2 + 2 = 6.

   - **If you chose B:** That is one group, $^1/_5$ of 10. The top number is 3, so take 3 groups.
   - **If you chose C:** You took 5 away from 10. The bottom number says how many equal groups to make.

8. **4 groups**

   The bottom number is 4. So Tariq makes 4 equal groups, with 2 stones in each.

   - **If you chose B:** The top number says how many groups to take. The bottom number says how many to make.
   - **If you chose C:** There are 2 stones in each group. But Tariq makes 4 groups.

9. **$^1/_3$**

   The 6 stones are in 3 equal groups. Priya takes 1 group, so she takes $^1/_3$.

   - **If you chose A:** You used the 2 stones in a group. The bottom number counts the groups, which is 3.
   - **If you chose C:** That is one stone out of 6. Priya took a whole group of stones.

M2.4.5. The number line, past 1

1. **Point B**

   B is 5 steps from 0, and each step is $^1/_4$. So B is $^5/_4$.

   - **If you chose A:** A is 3 steps from 0. That is $^3/_4$, before 1.
   - **If you chose C:** C is 7 steps from 0. That is $^7/_4$, past 1 but too far.

2. **Past 1**

   4 steps of $^1/_4$ reach 1. $^6/_4$ has 6 steps, so it is past 1.

   - **If you chose A:** You may think a fraction is always small. Here the top, 6, is bigger than the bottom, 4.
   - **If you chose B:** Only $^4/_4$ is at 1. $^6/_4$ is two more steps along.

3. **4/3**

   1 is 3 steps from 0. One more step makes 4 steps. Each step is $^1/_3$, so P is $^4/_3$.

4. **3/4**

   The bread has 4 equal parts, so the bottom is 4. She ate 3, so the top is 3.

   - **If you chose B:** That is one part. She ate 3 parts. Go to M2.4.3.
   - **If you chose C:** The bottom number is how many equal parts the whole has. The whole has 4. Go to M2.4.3.

5. **Mark 0, 1 and 2. Mark the middle of each whole. Count 3 half steps from 0. You land on $^{3}/_{2}$, halfway between 1 and 2.**

   Each step is $^{1}/_{2}$. Two steps reach 1, and one more step makes $^{3}/_{2}$.

6. **$^{5}/_{2}$**

   Each step is $^{1}/_{2}$. Q is 5 steps from 0, so Q is $^{5}/_{2}$.

   - **If you chose A:** The bottom number is how many steps make one whole. That is 2. The top number counts the steps from 0.
   - **If you chose C:** You counted the tick at 0 as a step. Count the steps after 0.

7. **$^{9}/_{6}$**

   6 steps of $^{1}/_{6}$ reach 1. $^{9}/_{6}$ has 9 steps, so it is past 1.

   - **If you chose A:** 5 steps of $^{1}/_{8}$ do not reach 1. You need 8 steps.
   - **If you chose B:** 6 steps of $^{1}/_{6}$ reach 1 exactly. It is at 1, not past it.

8. **5 steps**

   The top number counts the steps from 0. $^{5}/_{3}$ is 5 steps of $^{1}/_{3}$.

   - **If you chose B:** There are 3 steps in each whole. Kofi went past 1, so he counted more than 3.
   - **If you chose C:** There are 2 steps past 1. Kofi counted every step from 0.

9. **Yes. The top is bigger than the bottom.**

   4 steps of $^{1}/_{4}$ reach 1. $^{7}/_{4}$ has 7 steps, so it is past 1.

   - **If you chose A:** A fraction can be more than 1. 7 steps of $^{1}/_{4}$ go past the 4 steps that reach 1.
   - **If you chose C:** Only 4 quarters make one whole, $^{4}/_{4}$. $^{7}/_{4}$ is 3 more steps along, so it is past 1.

M2.4.6. Same amount, different names

1. **2/4**

   Cut each half in 2. The bottom doubles from 2 to 4. The top doubles from 1 to 2.

   - **If you chose B:** One part of 4 is less than a half. A half is 2 parts of 4.
   - **If you chose C:** Adding 1 to both numbers changes the amount. Double both numbers.

2. **2 of 6, which is 2/6**

   Each third becomes 2 parts, so the bar has 6 parts. The shaded third becomes 2 parts.

3. **Yes. Both name the same amount.**

   3 parts is half of 6 parts. So $^{3}/_{6}$ is the same amount as $^{1}/_{2}$.

   - **If you chose B:** Bigger numbers do not mean more. Fold a strip in 6 and shade 3. Compare it with a half.

4. **1/4**

   The bread has 4 equal parts, so the bottom is 4. She ate 1 part.

   - **If you chose B:** You may have counted the parts left. The bottom number is all the equal parts. The bread has 4. Go to M2.4.2.
   - **If you chose C:** The bottom number is how many equal parts. The top is how many you take. Go to M2.4.2.

5. **6**

   Cut each of the 4 parts in 2. Now there are 8 parts. Each shaded part becomes 2, so 3 parts become 6.

6. **No. They are equal. The shaded parts have the same length.**

   Both shaded parts end in the same place. So they are the same amount.

   - **If you chose A:** Look at the length, not the number of parts. The parts are smaller, so 2 of them match 1 half.
   - **If you chose C:** The parts are smaller, but there are more of them. Together they fill the same length.

7. **$^{2}/_{6}$**

   Cut each third in 2. The bottom doubles from 3 to 6. The top doubles from 1 to 2.

   - **If you chose A:** That is one part of 6, a smaller amount. When the bottom doubles, the top must double too.
   - **If you chose B:** Adding 1 to both numbers changes the amount. $^{2}/_{4}$ is the same as $^{1}/_{2}$.

8. **$^{6}/_{8}$**

   The bar has 8 parts now. Each shaded part becomes 2, so 3 shaded parts become 6.

   - **If you chose B:** You doubled the bottom number but not the top. Each shaded part becomes 2, so 3 shaded parts become 6.
   - **If you chose C:** Adding 1 to both numbers changes the amount. When each part is cut in 2, both numbers double.

9. **$^{2}/_{6}$ and $^{1}/_{3}$**

   Cut each third in 2 and $^{1}/_{3}$ becomes $^{2}/_{6}$. The amount stays the same.

   - **If you chose A:** Both have 1 part, but $^{1}/_{8}$ is a smaller part. They are not the same amount.
   - **If you chose C:** Adding 1 to both numbers changes the amount. $^{3}/_{4}$ is more than $^{2}/_{3}$.

M2.4.7. Which is bigger

1. **5/8**

   Both are eighths, so the parts are the same size. 5 of them are more than 3 of them.

   - **If you chose A:** The parts are the same size. 5 parts is more than 3 parts. Go to M2.4.3.

2. **1/6**

   Sixths are smaller parts than thirds. So $^{1}/_{6}$ is less.

   - **If you chose A:** You may think 6 is bigger, so $^{1}/_{6}$ is more. More parts means smaller parts. Go to M2.4.2.

3. **2/6**

   Cut each third in 2. The bottom doubles from 3 to 6. The top doubles from 1 to 2.

   - **If you chose B:** Cut each third in 2. The shaded third becomes 2 parts, not 1. Go to M2.4.6.
   - **If you chose C:** Changing only the top changes the amount. Double both numbers. Go to M2.4.6.

4. **1/8, 1/4, 1/2**

   All three have the top 1. The bigger the bottom number, the smaller the part. So $^1/_8$ is the least and $^1/_2$ is the most.

5. **Kofi**

   Both ate 3 parts. A quarter is bigger than an eighth, so Kofi ate more.

   - **If you chose B:** Each has 3 parts. But eighths are smaller than quarters. Go to M2.4.2.
   - **If you chose C:** Both have 3 parts, but the parts are not the same size. Go to M2.4.2.

6. **Ravi**

   Both are eighths, so the parts are the same size. 5 of them are more than 3.

   - **If you chose B:** Bigger number, smaller part is a rule for the bottom number. Here both bottoms are 8, so the top decides. 5 parts are more than 3.
   - **If you chose C:** Both are eighths. But Ravi has 5 of them and Amina has 3.

7. **$^3/_5$**

   Both have 3 parts. Fifths are bigger parts than tenths, so $^3/_5$ is more.

   - **If you chose A:** Tenths are smaller than fifths. 3 small parts are less than 3 bigger parts.
   - **If you chose B:** Both have 3 parts, but the parts are not the same size. Fifths are bigger than tenths.

8. **No. More parts in the whole means smaller parts.**

   Eighths are smaller parts than quarters. So $^1/_8$ is less than $^1/_4$.

   - **If you chose A:** That works for whole numbers. In a fraction, the bottom number counts the parts. More parts are smaller.
   - **If you chose C:** The top is the same, but the bottom numbers are not. An eighth is a smaller part than a quarter.

9. **$^3/_8$, $^3/_5$, $^3/_4$**

   All three have 3 parts. The bigger the bottom number, the smaller the part. So $^3/_8$ is the least.

   - **If you chose B:** That puts the smallest bottom number first. Eighths are the smallest parts, so $^3/_8$ is the least.
   - **If you chose C:** Each has 3 parts, but the parts are not the same size. Quarters, fifths and eighths differ.

M2.4.8. Adding and taking away parts

1. **5/7**

   Add the tops. 2 + 3 = 5. Keep the bottom, 7.

   - **If you chose B:** You added the bottoms too. The parts stay sevenths, so the bottom stays 7.
   - **If you chose C:** You may have multiplied the tops. Add them. 2 + 3 = 5.

2. **3/6**

   5 − 2 = 3. Keep the bottom, 6. The answer $^3/_6$ is also one half.

3. **5/4**

   3 + 2 = 5, and the bottom stays 4. So he lands on $^5/_4$. That is past 1.

4. **4 stones**

   Share 12 stones into 3 equal groups. Each group has 4. 4 + 4 + 4 = 12.

5. **4/8 litre**

   7 − 3 = 4. The bottom stays 8. $^4/_8$ is the same as one half litre.

6. **$^{4}/_{6}$**

   Add the tops. 1 + 3 = 4. Keep the bottom, 6.

   - **If you chose A:** You added the bottoms too. All the parts are sixths, so the bottom stays 6.
   - **If you chose C:** That is only the second part she ate. In all means add both parts. 1 + 3 = 4.

7. **$^{3}/_{8}$**

   Take away the tops. 5 − 2 = 3. Keep the bottom, 8.

   - **If you chose A:** You added. Giving away is taking away, so do 5 − 2.
   - **If you chose B:** That is the part Yusuf gave away. The question asks what is left.

8. **$^{2}/_{6}$**

   A hop to the left takes away. 5 − 3 = 2. The bottom stays 6.

   - **If you chose B:** You hopped to the right. A hop to the left takes away.
   - **If you chose C:** That is the size of the hop. Where she lands is 5 − 3 = 2 sixths.

9. **$^{3}/_{4} - {}^{2}/_{4} = {}^{1}/_{4}$**

   Mei starts at $^{3}/_{4}$ and hops left $^{2}/_{4}$. 3 − 2 = 1. The bottom stays 4.

   - **If you chose A:** The hop goes to the left, so it takes away. A hop to the right adds.
   - **If you chose C:** You wrote the size of the hop. 3 − 2 = 1, so she lands on $^{1}/_{4}$.

About this book

For learners who can count to 100, add and take away, and share things into equal groups. In many schools this fits children of about 8 to 10, but older learners who missed fractions can start here too. Each chapter takes one or two short sessions. A learner can work alone: every exercise has its answer and the reason for it.

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